The Open University

M208
Diagnostic quiz

Am I ready to start M208?

The diagnostic quiz below is designed to help you to answer this question. This document also contains some advice on preparatory work that you may find useful before starting M208 (see below and page 7).

The topics which are included in this quiz are those that we expect you to be reasonably familiar with before you start the course because we use the results and techniques without comment in M208. There are other topics such as complex numbers, groups, matrices and vectors, which it would be helpful to have met, but these topics are covered again from scratch in the course, so we have not included them in this quiz.

We suggest that you try this quiz first without looking at any books or using a calculator, and only look at a book when you are stuck. You will not necessarily remember everything; for instance, for the calculus questions you may need to look up a set of rules for differentiation or integration or a table of standard derivatives or integrals. This is perfectly all right, as such tables are provided in the Handbook for M208. You only need to check that you are able to use them.

If you can complete the quiz in a reasonable time, with only the occasional need to look at other material, then you should be well prepared for M208. If you found some areas that you remember having met before but need to do some more work on, then you should consider the suggestions for additional study on page 7.

M208 is a very rewarding course, covering a wide range of pure mathematics. The better prepared you are for it the more time you will have to enjoy the mathematics, and the greater your chance of success.

Try the questions now, and then see the notes on page 7 of this document to see if you are ready for M208.

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1.1

Diagnostic quiz questions

Give numerical answers in exact form, for example, 2π and $\sqrt{3}$.

1. Solve each of the following equations for x.

(a) $2x + 7 = 13$ (b) $3(x + 3) = 7(x - 1)$

(c) $\dfrac{3}{1 - x} = \dfrac{2}{2 + x}$ (d) $3x^2 - x = 0$

(e) $2x^2 - 5x - 3 = 0$ (f) $2x^2 + 7x + 4 = 0$

2. Solve the following simultaneous equations for x and y:

$$2x - 3y = 4,$$
$$x + 2y = 9.$$

3. (a) Make u the subject of the equation

$$t^2 = \frac{2(s - ut)}{a}.$$

(b) Make x the subject of the equation

$$\sqrt{\frac{x - 2}{x + 3}} = t.$$

4. Simplify each of the following expressions.

(a) x^3x^4

(b) x^2/x^5

(c) $\left(x^3\right)^2$

(d) $4^{1/2}$

(e) $\left(e^{-2x} \times e^{3x}\right)^2$

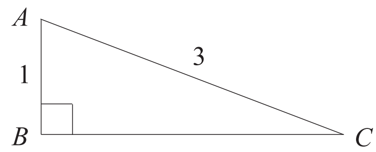
5. Simplify each of the following expressions.

(a) $t(2t + 3) - 2t(1 - 3t)$

(b) $\dfrac{21a^5b^7}{49a^3b^{10}}$

(c) $\dfrac{12p^2 + 20p + 8}{6p^2 + 7p + 2}$

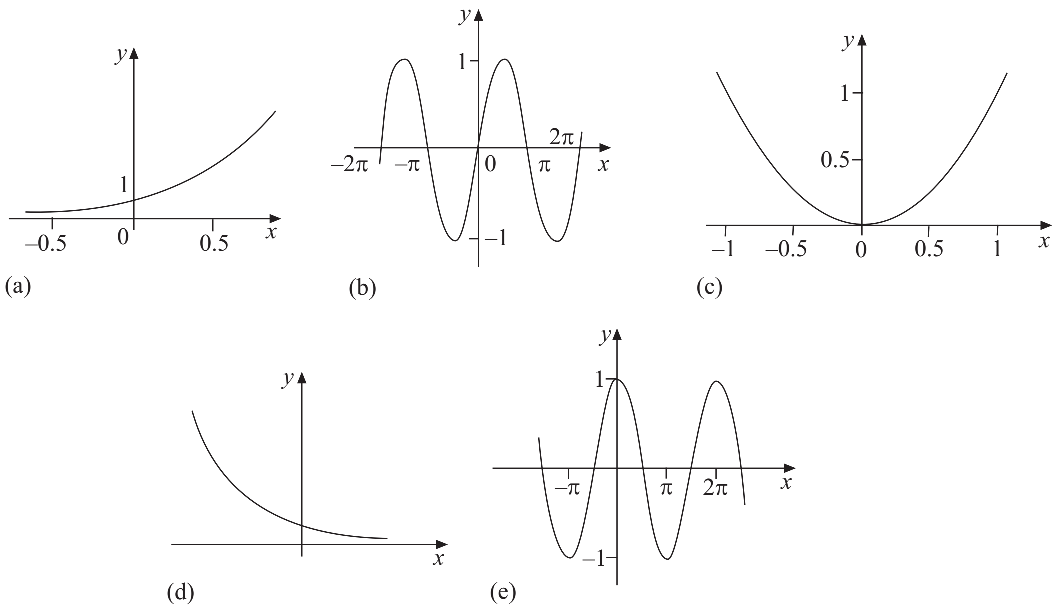
6. In a right-angled triangle ABC with right-angle at B, AB has length 1 and AC has length 3. What is the length of BC?



7. What is the equation of the straight line through the points $(2, 1)$ and $(4, 5)$? What is the gradient (or slope) of this line?

8. Five graphs are given in parts (a)–(e) of the figure below. Each graph is that of one of the functions (i)–(v). Match each graph with the appropriate function.

Functions: (i) $f(x) = e^{-2x}$; (ii) $f(x) = e^{2x}$; (iii) $f(x) = \sin x$; (iv) $f(x) = \cos x$; (v) $f(x) = x^2$.



9. What are the exact values of

$\cos\left(\frac{1}{2}\pi\right)$, $\sin\left(\frac{1}{4}\pi\right)$, $\tan\left(\frac{1}{4}\pi\right)$, $\cos\left(\frac{1}{3}\pi\right)$,

$\sin\frac{1}{3}\pi$, $\cos\left(\frac{1}{6}\pi\right)$, $\sin\left(\frac{1}{6}\pi\right)$?

10. What are the exact values of

$\cos\left(\frac{3}{4}\pi\right)$, $\sin\left(\frac{7}{4}\pi\right)$, $\tan\left(\frac{5}{4}\pi\right)$?

11. If $\sin\theta = \frac{3}{5}$, what are the possible values of $\cos\theta$?

12. Differentiate the following functions with respect to x.

(a) $f(x) = x^4 + 5x^3 - x^2 + 2x - 1$

(b) $f(x) = \sin x$

(c) $f(x) = 3e^{2x}$

13. Differentiate the following functions with respect to x.

(a) $f(x) = (x^3 + 3)\cos x$ (b) $f(x) = e^x(x^2 + 5x - 3)$

(c) $f(x) = \dfrac{5}{x^2 - 1}$ (d) $f(x) = \dfrac{x^2 - x + 1}{2x + 3}$

14. (a) Find $f'(x)$, where $f(x) = e^{3\sin x}$.

(b) Find $f'(x)$, where $f(x) = \cos\left(3x^2 + 2x - 6\right)$.

15. Evaluate each of the following integrals.

(a) $\displaystyle\int (2x^3 + 5)\,dx$ (b) $\displaystyle\int_0^{\pi/2} \cos(5t)\,dt$

16. (a) Use integration by parts to evaluate $\displaystyle\int x\sin x\,dx$.

(b) Use integration by substitution to evaluate $\displaystyle\int x^2 e^{4x^3}\,dx$.

Diagnostic quiz answers

1. We solve these equations by rearranging them into equivalent simpler forms.

(a) $2x + 7 = 13$

$2x = 6$

$x = 3$

(b) $3(x + 3) = 7(x - 1)$

$3x + 9 = 7x - 7$

$3x - 7x = -7 - 9$

$-4x = -16$

$x = 4$

(c) $\dfrac{3}{1-x} = \dfrac{2}{2+x}$

$3(2 + x) = 2(1 - x)$

$6 + 3x = 2 - 2x$

$5x = -4$

$x = -\frac{4}{5}$

(d) $3x^2 - x = 0$

$x(3x - 1) = 0.$

Hence $x = 0$ or $3x - 1 = 0$.
So $x = 0$ or $x = \frac{1}{3}$,
so the solutions are 0 or $\frac{1}{3}$.

(e) $2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0.$

Hence $2x + 1 = 0$ or $x - 3 = 0$.
So $x = -\frac{1}{2}$ or $x = 3$,
so the solutions are $-\frac{1}{2}$ and 3.

(f) Using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 2$, $b = 7$ and $c = 4$ gives

$$x = \frac{-7 \pm \sqrt{7^2 - 4.2.4}}{4}$$

$$= \frac{-7 \pm \sqrt{49 - 32}}{4}$$

$$= \frac{-7 \pm \sqrt{17}}{4},$$

so the solutions are $\dfrac{-7 + \sqrt{17}}{4}$ and $\dfrac{-7 - \sqrt{17}}{4}$.

2. $2x - 3y = 4,$

$x + 2y = 9.$

Multiplying the second equation by 2 gives

$2x - 3y = 4$

$2x + 4y = 18.$

Subtracting the first equation from the second gives

$7y = 14,$

hence $y = 2$.
Substituting for y into the second original equation gives

$x + 4 = 9,$

hence $x = 5$.

So the solution is $x = 5$, $y = 2$.

3. (a) $t^2 = \dfrac{2(s - ut)}{a}$

$\Leftrightarrow at^2 = 2(s - ut) = 2s - 2ut$

$\Leftrightarrow 2ut = 2s - at^2$

$\Leftrightarrow \quad u = \dfrac{2s - at^2}{2t}$

(b) $\sqrt{\dfrac{x-2}{x+3}} = t$

$\Rightarrow \dfrac{x-2}{x+3} = t^2,$

so $x - 2 = t^2(x + 3) = t^2x + 3t^2$.

Rearranging this equation gives

$x(1 - t^2) = 2 + 3t^2$

$x = \dfrac{2 + 3t^2}{1 - t^2}.$

4. (a) $x^3x^4 = x^{3+4} = x^7$

(b) $x^2/x^5 = x^{2-5} = x^{-3}$

(c) $(x^3)^2 = x^{3\times 2} = x^6$

(d) $4^{1/2} = \sqrt{4} = 2$

(e) $\left(e^{-2x} \times e^{3x}\right)^2 = \left(e^{-2x+3x}\right)^2 = (e^x)^2 = e^{2x}$

5. (a) $t(2t + 3) - 2t(1 - 3t)$

$= 2t^2 + 3t - 2t + 6t^2$

$= 8t^2 + t$

(b) $\dfrac{21a^5b^7}{49a^3b^{10}} = \dfrac{21}{49} \times \dfrac{a^5}{a^3} \times \dfrac{b^7}{b^{10}}$

$= \dfrac{3}{7} \times \dfrac{a^2}{b^3}$

$= \dfrac{3a^2}{7b^3}$

(c) $$\frac{12p^2+20p+8}{6p^2+7p+2} = \frac{4(3p^2+5p+2)}{(3p+2)(2p+1)} = \frac{4(3p+2)(p+1)}{(3p+2)(2p+1)} = \frac{4(p+1)}{(2p+1)}$$

6. By Pythagoras' Theorem,

$$AC^2 = AB^2 + BC^2$$
$$9 = 1 + BC^2,$$

so

$$BC^2 = 9 - 1 = 8$$
$$BC = \sqrt{8} = 2\sqrt{2}.$$

7. The general equation of a straight line is $y = mx + c$. Substituting the coordinates of P and Q into this equation gives

$$1 = 2m + c,$$
$$5 = 4m + c.$$

Subtracting the first equation from the second gives

$$4 = 2m,$$

that is, $m = 2$.

Using the first equation gives

$$1 = 4 + c,$$

that is, $c = -3$.

So the equation of the line is $y = 2x - 3$. The gradient of the line is 2, as $m = 2$. (There are other ways of doing this question.)

8. The matching is as follows:

(a) (ii); (b) (iii); (c) (v); (d) (i); (e) (iv).

(Graphs like these are revised in the first unit of M208.)

9. $\cos\left(\frac{1}{2}\pi\right) = 0$, $\sin\left(\frac{1}{4}\pi\right) = \frac{1}{\sqrt{2}}$, $\tan\left(\frac{1}{4}\pi\right) = 1$,

$\cos\left(\frac{1}{3}\pi\right) = \frac{1}{2}$, $\sin\left(\frac{1}{3}\pi\right) = \frac{\sqrt{3}}{2}$, $\cos\left(\frac{1}{6}\pi\right) = \frac{\sqrt{3}}{2}$,

$\sin\left(\frac{1}{6}\pi\right) = \frac{1}{2}$.

10. $\cos\left(\frac{3}{4}\pi\right) = -\cos\left(\frac{1}{4}\pi\right) = -\frac{1}{\sqrt{2}}$,

$$\sin\left(\tfrac{7}{4}\pi\right) = \sin\left(2\pi - \tfrac{1}{4}\pi\right) = \sin\left(-\tfrac{1}{4}\pi\right) = -\sin\left(\tfrac{1}{4}\pi\right) = -\frac{1}{\sqrt{2}},$$

$\tan\left(\frac{5}{4}\pi\right) = \tan\left(\frac{1}{4}\pi\right) = 1$.

11. We have the identity $\sin^2\theta + \cos^2\theta = 1$.

So, if $\sin\theta = \frac{3}{5}$, then

$$\left(\tfrac{3}{5}\right)^2 + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \left(\tfrac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25},$$

so $\cos\theta = \pm\frac{4}{5}$.

12. (a) $f'(x) = 4x^3 + 5 \times 3x^2 - 2x + 2 = 4x^3 + 15x^2 - 2x + 2$

(b) $f'(x) = \cos x$

(c) $f'(x) = 3 \times 2e^{2x} = 6e^{2x}$

(There is a table of standard derivatives in the Handbook for M208.)

13. (a) We use the Product Rule. If $f(x) = g(x)\,h(x)$, then

$$f'(x) = g'(x)h(x) + g(x)h'(x),$$

with $g(x) = x^3 + 3$ and $h(x) = \cos x$.

So

$$f'(x) = 3x^2\cos x + (x^3+3)(-\sin x) = 3x^2\cos x - (x^3+3)\sin x.$$

(b) We use the Product Rule:

$$f'(x) = e^x(x^2+5x-3) + e^x(2x+5) = e^x(x^2+7x+2).$$

(c) We apply the Composite Rule to the function

$$f(x) = \frac{5}{(x^2-1)} = 5(x^2-1)^{-1}.$$

So

$$f'(x) = 5(-1)(x^2-1)^{-2}2x = \frac{-10x}{(x^2-1)^2}.$$

(d) We use the Quotient Rule.

If $f(x) = \dfrac{g(x)}{h(x)}$, then $f'(x) = \dfrac{h(x)g'(x) - h'(x)g(x)}{(h(x))^2}$.

So, since

$$f(x) = \frac{x^2-x+1}{2x+3},$$
$$g(x) = x^2 - x + 1,$$
$$h(x) = 2x + 3;$$

then

$$f'(x) = \frac{(2x+3)(2x-1) - 2(x^2-x+1)}{(2x+3)^2} = \frac{(4x^2+4x-3) - (2x^2-2x+2)}{(2x+3)^2} = \frac{2x^2+6x-5}{(2x+3)^2}.$$

14. We use the Composite Rule for both parts.

(a) $f'(x) = e^{3\sin x} \times 3\cos x = 3e^{3\sin x}\cos x$

(b) $$\begin{aligned} f'(x) &= -\sin\left(3x^2 + 2x - 6\right) \times (6x + 2) \\ &= -(6x + 2)\sin(3x^2 + 2x - 6) \end{aligned}$$

15. (a) This is an indefinite integral.

$$\begin{aligned} \int (2x^3 + 5)dx &= 2 \times \tfrac{1}{4}x^4 + 5x + c \\ &= \tfrac{1}{2}x^4 + 5x + c, \end{aligned}$$

where c is an arbitrary constant.

(b) This is a definite integral.

$$\begin{aligned} \int_0^{\pi/2} \cos(5t)\, dt &= \left[\tfrac{1}{5}\sin(5t)\right]_0^{\pi/2} \\ &= \tfrac{1}{5}\sin\left(\tfrac{5}{2}\pi\right) - \tfrac{1}{5}\sin(0) \\ &= \tfrac{1}{5} \times 1 - \tfrac{1}{5} \times 0 = \tfrac{1}{5} \end{aligned}$$

16. (a) The general formula for integration by parts is

$$\int f(x)g'(x)\, dx = f(x)g(x) - \int f'(x)g(x)\, dx.$$

Let $f(x) = x$ and $g'(x) = \sin x$. Then $f'(x) = 1$ and $g(x) = -\cos x$, so

$$\begin{aligned} \int x\sin x\, dx &= -x\cos x - \int 1(-\cos x)\, dx \\ &= -x\cos x + \int \cos x\, dx \\ &= -x\cos x + \sin x + c, \end{aligned}$$

where c is an arbitrary constant.

(b) Integration by substitution uses the formula

$$\int f(g(x))g'(x)\, dx = \int f(u)\, du,$$

where $u = g(x)$.

Let $u = 4x^3$. Then $g'(x) = 12x^2$, so

$$\begin{aligned} \int x^2 e^{4x^3}\, dx &= \frac{1}{12}\int e^{4x^3} 12x^2\, dx \\ &= \frac{1}{12}\int e^u du \\ &= \frac{1}{12}e^u + c = \frac{1}{12}e^{4x^3} + c, \end{aligned}$$

where c is an arbitrary constant.

What can I do to prepare for M208?

Now that you have finished the quiz, how did you get on? The information below should help you to decide what, if anything, you should do next.

Most of this quiz was unfamiliar to me. I don't think I have met any of the topics before.	→ Perhaps you should study another course such as MU120 or MST121. Contact a Regional Adviser to discuss this.
Some of these topics are familiar to me, but not all of them.	→ Consider taking MST121 and MS221. (Or MS221 if you have already taken MST121.)
I have met all these topics before, but I am a bit rusty on some, and had to look up how to do a few of them.	→ You will benefit from doing some revision before starting M208 – see below for some possible sources of help.
I can do all of these questions.	→ You are well prepared for M208. You can study it with confidence.

What resources are there to help me revise for M208?

If you have studied MST121, MS221 or M101 before, then you could use parts of these to revise for M208.

If you need to brush up topics like algebra, trigonometry and calculus, then you may find some of the textbooks designed for A-level students useful. Alternatively, there are some suitable books in the Teach Yourself series published by Hodder and Stoughton.

There are also two books called *Countdown to Mathematics*, Volumes 1 and 2, by Lynne Graham and David Sargent, which have plenty of examples for practice. They are published by Addison Wesley, and their ISBN numbers are 201 13730 5 for Volume 1 and 201 13731 3 for Volume 2.

In addition, there are many websites that offer revision materials in mathematics that you may find useful. However the sites listed below are not Open University websites, so we cannot guarantee that they will continue to be available.

Quick revision notes and exercises

http://www.bbc.co.uk/scotland/education/bitesize/higher/maths/
[Accessed 2 May 2006]
This is a resource developed by BBC Scotland for students studying for the Scottish Higher Examination in Mathematics. It covers algebra, trigonometry and elementary calculus.

The Maths Support Centre

http://www.mathcentre.ac.uk/students.php [Accessed 2 May 2006]
This site has teach-yourself booklets, summary sheets, revision booklets, online exercises and video tutorials on a wide range of topics to help you to develop the mathematical skills needed for M208.

Calculus on the web

http://cow.math.temple.edu/ [Accessed 2 May 2006]
This is an interactive site with examples on algebra, calculus, linear algebra, abstract algebra and number theory.

Other resources

You can also look for other resources on the web. If you go to the Open University Library's ROUTES service at http://routes.open.ac.uk/ [Accessed 1 May 2006] and type in MST121 in the search box, you will find various websites which you may enjoy.

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